# Variability in Stepping Direction Explains the Veering Behavior of Blind Walkers 

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#### Abstract

Walking without vision results in veering, an inability to maintain a straight path that has important consequences for blind pedestrians. In this study, the authors addressed whether the source of veering in the absence of visual and auditory feedback is better attributed to errors in perceptual encoding or undetected motor error. Three experiments had the following results: No significant differences in the shapes of veering trajectories were found between blind and blindfolded participants; accuracy in detecting curved walking paths was not correlated with simple measures of veering behavior; and explicit perceptual cues to initial walking direction did not reduce veering. The authors present a model that accounts for the major characteristics of participants' veering behavior by postulating 3 independent sources of undetected motor error: initial orientation, consistent biases in step direction, and, most important, variable error in individual steps.


Keywords: veering, walking, locomotion, motor noise, blind mobility

In the context of human locomotion, veering is any deviation from an intended path. When walking an intended straight line, veering is the lateral deviation from that line. Veering by human pedestrians becomes evident when visual targeting cues are absent, as in cases of blindness or severely reduced visibility (e.g., walking in the dark or in a blizzard). For blind people, veering can be an everyday problem, potentially threatening the safety of blind people when they are crossing a street at a busy intersection. Nonvisual environmental cues such as wind direction, slope of the ground plane, or directional acoustic cues can help blind pedestrians accurately navigate across spaces. Unfortunately, reliable nonvisual cues are not always available. In the absence of such cues, our research and the findings of others (to be reviewed in this article) show that blind people veer away from intended straight pathways. The purpose of the present study was to answer the question: What causes this veering behavior? Our research suggests that a simple explanation, unperceived motor noise at the level of individual steps, may explain the veering behavior of blind pedestrians and sighted pedestrians who are blindfolded.

Some early proposals for the explanation of veering behavior, reviewed by Guth and LaDuke (1994) and by Cratty (1965,

[^0]1971), included dominance or difference in leg strength or leg length, biomechanical asymmetries, and evolutionary hypotheses. Cratty (1965), as well as researchers in later studies reviewed by Guth and LaDuke (1994), focused on blind pedestrians and were motivated by interests in enhancing mobility skills of blind people. Cratty (1965) noted that researchers in early studies showed the presence of veering in blind walkers, but magnitudes were not accurately measured or reported. Cratty (1965) also recognized that prior empirical studies of human veering were flawed either because of inaccurate or nonobjective measures. Later, Guth and LaDuke (1994) explained that in the majority of prior empirical studies, the researchers used group mean measures of walking performance that were insufficient for accurate characterization of individual veering behavior. As emphasized by Guth and LaDuke (1994), these group measures did not describe individuals' distinct characteristics. The authors argued that researchers should consider both directional bias and trial-to-trial variability to capture the veering patterns of individuals. Earlier studies were problematic because researchers collapsed individual data across trials and days. Furthermore, these hypotheses predicted consistent rather than variable patterns of walking.

Guth and LaDuke (1995) focused their analysis on the lateral offsets from a nominal straight line (veer) at the end of a nonvisual walking trial. We took their analysis further by recognizing that different walking trajectories can result in the same lateral offset. For instance, one person might walk a perfectly straight line but end up with an offset from the intended straight line because of an initial orientation error. Another person might begin straight and curve away from the intended pathway, possibly due to a directional stepping bias, but end up in the same final location as that of the first person. Because these trajectory differences may arise from distinct causes, it is informative for researchers to measure and analyze individual trajectories and their patterns rather than just their endpoints.

A pedestrian's ability to walk a straight line depends on the availability and quality of sensory information about walking direction (Loomis \& Beall, 1998; Philbeck, Loomis, \& Beall, 1997; Rieser, Ashmead, Talor, \& Youngquist, 1990) and on the capacity to execute movements in an intended direction. In addition to vision, sources of sensory information used during navigation include acoustic, tactile, vestibular, proprioceptive and kinesthetic cues. Vision and audition can be used to maintain a constant heading toward a distant target. Tactile cues may be useful for following a prescribed route (e.g., a gravel path). Vestibular, proprioceptive or kinesthetic cues may be useful in detecting rotation away from a straight path (Chance, Gaunet, Beall, \& Loomis, 1998). Evidence from a study of patients with vestibular pathologies (Cohen, 2000) suggested that normal veering could be related to sensitivity in the vestibular system. Finally, interfering sensory cues may influence veering. For instance, Millar (1999) showed that extraneous auditory noise and influences on postural signals (such as a handbag in one hand) can result in greater veering.

Even in the absence of any sensory input, it is possible that a human (or robot) could proceed along a fairly straight path. This notion motivates consideration of the precision of the motor system in executing movements in a constant linear direction. Imagine, for example, a robotic vehicle with no sensors of any kind. If it is placed on a flat surface with isotropic frictional characteristics and if there are no deflecting forces such as wind, the ability of the vehicle to travel in a straight line would depend on the mechanical design and characteristics of the motor. Imprecision could manifest as a predisposition for the vehicle to take the wrong heading, curve toward the left or right, or zigzag erratically. Similarly, it is possible that when people are deprived of adequate perceptual input for maintaining an intended path, motor or biomechanical limitations may limit accuracy and may account for veering behavior. More generally, the ability of a human or robot to travel a straight line would depend on motor precision, the quality of sensory feedback, and the information loop connecting these two.

Although blind walkers often use acoustic or tactile cues during walking, those cues are often uninformative and sometimes misleading. For that reason, we asked what factors limit straight-line walking in the absence of visual, acoustic, or tactile cues.

In Experiment 1, we assessed the veering behavior of blind walkers and sighted walkers who were blindfolded. They repeatedly attempted to walk a straight line after being physically aligned to the desired walking direction. Two-dimensional walking trajectories were sampled every 1.52 m for a total of 9.14 m . These measurements and methods were similar to those employed by Guth and LaDuke (1995), providing a replication of their work and a baseline for comparison and modeling in subsequent parts of our study.

In Experiment 2, we asked whether sensory limitations (vestibular, proprioceptive, or kinesthetic) account for the magnitude of veering errors. We did this by measuring participants' thresholds for detecting the curvature of walking paths (similar to Cratty, 1965). We reasoned that if the ability to walk a straight line is limited by the sensory capacity to detect deviations from straightness (i.e., the ability to detect curved paths), then people with poor curvature detection should exhibit greater veering behavior.

In Experiment 3, we asked how veering is affected by three different initial orientation conditions, including physical align-
ment used in Experiment 1, and two conditions with explicit perceptual pointers. In this experiment, we tested whether veering behavior is due to inadequate perceptual information of initial orientation. The results address the merits of providing explicit perceptual cues, such as physical pointers that indicate walking direction, which may be of practical value in blind mobility.

Finally, in the modeling section, a simple random-walk model replicated the types of trajectories and statistical properties of human performance when walking without vision. We found that this model can simulate qualitative and quantitative characteristics of human veering behavior. The agreement found between model and human behavior suggests that motor noise may explain human veering in the absence of vision. We believe that motor noise occurs at the level of individual human steps. In other words, errors in individual steps can accumulate over time, causing the pattern of veering exhibited by blind walkers.

## Experiment 1

## Objective

How accurately do blind people and blindfolded sighted people walk a straight line without vision, and are there differences between these groups? Studies of blind versus sighted individuals suggest that people with different visual experience may have only slight differences in performance under a variety of different motor tasks, such as walking a maintained heading and replicating distance (Klatzky, Golledge, Loomis, Cicinelli, \& Pellegrino, 1995) and performing wayfinding tasks (Giudice, 2004). In walking a straight line, blind people might perform better because they are experienced in nonvisual locomotion and make better use of vestibular, proprioceptive, or kinesthetic feedback. Alternatively, sighted people may perform better because they constantly use visual feedback, including optical flow, to calibrate their motor responses (cf. Rieser, Hill, Talor, Bradfield, \& Rosen, 1992). Finally, sighted and blind pedestrians may perform equally well if performance is limited by nonperceptual factors such as intrinsic noisiness of the motor system.

## Method

Participants. Five blind and 5 sighted individuals participated (see Table 1). Three of the blind participants had slight residual vision, but all participants relied primarily on nonvisual cues for mobility. All blind participants used dog guides for day-to-day travel, but dogs were not used during the experiments.

Testing arrangements and procedure. Experiments were conducted in a $5.5 \mathrm{~m} \times 20.1 \mathrm{~m}$ room in the basement of Elliott Hall at the University of Minnesota. We used $1.9-\mathrm{cm}$ tape to mark an area on the floor that was $4.88-\mathrm{m}$ wide $\times 9.14 \mathrm{~m}$ long. Along the $9.14-\mathrm{m}$ pathway, a perpendicular line was taped at every $1.52-\mathrm{m}$ interval and marked with $0.30-\mathrm{m}$ hash marks (seen in Figure 1C). These markings were not detectable by the participants. The long edge of a $0.91 \mathrm{~m} \times 1.83 \mathrm{~m}$ rectangular table was placed at the beginning of the walking pathway and was oriented perpendicular to the desired line of travel.

Blind participants with residual vision (Participants P2, P4, and P5) and sighted participants wore blindfolds. Blind participants (P1 and P3) did not require blindfolding because they had no residual vision. All participants wore earmuffs, which minimized sound cues. The experimenter could communicate with participants via FM radio headphones placed inside the earmuffs. With earmuffs and blindfolds in place, there

Table 1
Experiment 1 and 2 Participants

| Participant | Age | Gender | Pathology | Onset | Vision level |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Blind |  |  |  |  |  |
| P1 | 48 | F | Glaucoma | Birth | No light perception |
| P2 | 27 | M | Leber's congenital amaurosis | Birth | OU: Light perception, 20/1500 |
| P3 | 36 | M | Diabetic retinopathy | 15 years of age | No light perception |
| P4 | 48 | F | Retrolental fibroplasia | Birth | OD: Reads large print; OU: <br> Light perception, < 20/400 |
| P5 | 29 | F | Microthalmus, clouded cornea | Birth | OD: Light perception, prosthetic; OS: Color/light perception |
| Sighted |  |  |  |  |  |
| P6 | 24 | M |  |  |  |
| P7 | 21 | M |  |  |  |
| P8 | 50 | F |  |  |  |
| P9 | 41 | M |  |  |  |
| P10 | 31 | F |  |  |  |

Note. $\mathrm{OU}=$ both eyes; $\mathrm{OD}=$ right eye; $\mathrm{OS}=$ left eye.
were no acoustic, tactile, or visual cues for direction. Trials were conducted in the center of the room, away from the walls, and participants were assured that they would be prevented from colliding with obstacles or walls.

Participants began each trial by physically aligning their backs against the edge of the table, using their hands to feel the table's straight edge. The experimenter made sure that participants straddled the beginning of the intended trajectory line (seen in Figure 1A). Participants were instructed to walk a straight line directly away from (perpendicular to) the table, at a comfortable speed, until the experimenter asked them to stop. Trials were terminated either when the participant crossed a goal line 9.14 m from the table or veered more than 2.44 m from the intended pathway. Trials for each participant were recorded over 3-4 days, with a minimum of 9 trials per day and 39-45 trials per person (see Table 2).

Trials were videotaped and reviewed for scoring. The data were based on measurements taken from video frames at the points where participants passed over each $1.52-\mathrm{m}$ line. Locations were visually estimated in the video frames, and deviations were recorded to the nearest $8-10 \mathrm{~cm}$ (see Figure 1).

Across the 10 participants, 420 trials were conducted. Four hundred trials were sampled at $1.52-\mathrm{m}$ intervals out to the $9.14-\mathrm{m}$ goal line. The remaining 20 trials were sampled in the same manner but not out to the goal line because participants veered more than 2.44 m from the intended pathway. Although fewer samples were recorded in these 20 trials, they were still included in the analysis.

Because individual trajectories were fairly smooth (i.e., not much zigzagging), a least-squares polynomial with linear and quadratic terms was fitted to each sampled trajectory. Second-order polynomials were chosen because neither straight lines nor parabolas alone satisfactorily fit the data.


Figure 1. Example of one trial and polynomial fit. A unique second-order polynomial was fitted for each trial. Panels A, B, and C show video frames of a single trial at $0 \mathrm{~m}, 4.57 \mathrm{~m}, 9.14 \mathrm{~m}$, respectively. Panel D shows the least-squares polynomial fit.

Table 2
Walking Parameter Estimates (Experiment 1)

| Participant | Day | Initial orientation ( ${ }^{\circ}$ ) |  | Bias $\left({ }^{\circ}\right.$ ) |  | No. trials |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | M | $S D$ | M | $S D$ |  |
| P1 | 1 | -1.06 | 1.03 | -0.67 | 1.84 | 10 |
|  | 2 | 1.69 | 2.07 | 0.29 | 1.70 | 11 |
|  | 3 | 2.56 | 2.19 | 0.27 | 0.93 | 12 |
|  | 4 | 3.00 | 2.63 | 0.17 | 2.44 | 11 |
| P2 | 1 | -4.91 | 1.93 | -0.12 | 1.50 | 10 |
|  | 2 | -1.95 | 4.54 | -0.015 | 0.74 | 10 |
|  | 3 | -7.66 | 2.73 | -0.003 | 0.77 | 10 |
|  | 4 | -1.24 | 3.97 | -0.058 | 1.43 | 11 |
| P3 | 1 | -1.09 | 2.23 | 0.44 | 1.26 | 10 |
|  | 2 | -0.85 | 2.41 | 0.86 | 1.03 | 10 |
|  | 3 | -2.15 | 2.56 | 0.55 | 1.43 | 10 |
|  | 4 | -3.00 | 1.52 | 0.22 | 0.58 | 10 |
| P4 | 1 | 0.11 | 3.08 | 0.98 | 1.59 | 10 |
|  | 2 | -1.62 | 2.38 | 0.014 | 1.73 | 10 |
|  | 3 | 3.86 | 2.68 | 0.69 | 1.61 | 11 |
|  | 4 | -1.36 | 2.82 | -0.21 | 1.37 | 10 |
| P5 | 1 | -4.19 | 1.96 | -0.62 | 1.34 | 11 |
|  | 2 | -0.80 | 2.45 | 0.24 | 1.57 | 10 |
|  | 3 | 1.59 | 2.17 | -0.28 | 1.07 | 11 |
|  | 4 | 1.97 | 1.67 | -0.40 | 0.95 | 10 |
| P6 | 1 | 5.33 | 1.77 | 1.63 | 1.12 | 12 |
|  | 2 | 5.87 | 1.58 | 1.17 | 1.55 | 12 |
|  | 3 | 1.83 | 1.25 | 0.25 | 1.06 | 10 |
|  | 4 | 3.37 | 1.77 | 0.46 | 0.88 | 11 |
| P7 | 1 | 0.52 | 2.66 | 0.64 | 1.65 | 11 |
|  | 2 | 0.28 | 2.32 | 0.33 | 1.99 | 9 |
|  | 3 | -0.22 | 2.00 | 0.52 | 1.34 | 10 |
|  | 4 | 0.80 | 1.53 | 0.66 | 0.72 | 10 |
| P8 | 1 | -2.57 | 1.70 | 0.51 | 1.60 | 12 |
|  | 2 | -4.32 | 3.25 | 0.20 | 1.15 | 12 |
|  | 3 | -4.42 | 3.27 | -0.54 | 1.05 | 11 |
|  | 4 | -2.12 | 2.05 | -0.002 | 1.01 | 10 |
| P9 | 1 | 1.32 | 1.67 | 0.39 | 1.69 | 14 |
|  | 2 | 3.57 | 2.14 | 0.13 | 1.28 | 13 |
|  | 3 | 2.23 | 2.75 | -0.05 | 1.36 | 14 |
| P10 | 1 | 0.82 | 1.90 | 0.15 | 2.03 | 10 |
|  | 2 | -2.10 | 2.07 | 0.82 | 0.94 | 10 |
|  | 3 | 2.41 | 2.96 | 0.65 | 1.96 | 11 |
|  | 4 | -3.44 | 2.21 | 0.11 | 1.21 | 10 |

We examined the properties of the linear and quadratic coefficients of these fits and the correlations between them.

## Results

Mean signed (constant) veering errors at 9.14 m across all trials for individual participants ranged from -0.67 m (left) to 1.74 m (right). See Table 3 and Figure 2B for details. Mean unsigned (absolute) veering errors (mean of the absolute value of the errors) at 9.14 m ranged from 0.45 m to 1.74 m (see Table 3).

Individual differences (between participants) and variability over days (within participants) were evident. Veering consistency was calculated as the ratio of signed endpoint variance $\delta_{\text {total }}^{2}$ (across days) to the average of the within-day variances $\delta_{i}^{2}$ for each participant: $\frac{\delta_{\text {total }}^{2}}{\frac{1}{n}\left(\delta_{1}^{2}+\delta_{2}^{2}+\cdots+\delta_{\text {ndays }}^{2}\right)}$ (see Figure 3). No clear differences were observed between the blind and sighted groups. A
two-factorial, group-by-day, nested analysis of variance (ANOVA) on deviations at 9.14 m indicated no effect of group (blind vs. sighted), $F(1,8)=0.15, p=.71$, but indicated a within-participant effect of day, $F(29,381)=13.93, p<.001$. A second two-factorial ANOVA showed significant within- and between-participants differences. This ANOVA was performed in addition to the first because a balanced design was required for observation of the effect of participants, days, and the interaction (which did not test for a group effect). Our analysis in this ANOVA excluded Participant P9 because he was tested over 3 days, whereas all other participants in this experiment were tested over 4 days: between-participants effect, $F(8,240)=52.23, p<$ .001; within-participants effect over days, $F(3,240)=5.96, p=$ .017. A significant interaction between participants and days was also found, $F(24,240)=25.10, p<.001$. These findings are consistent with the findings of Guth and LaDuke (1995) that showed individual and day-to-day variations in performance.

Analysis of trajectories showed that in many trials, there were statistically significant linear and quadratic contributions to the veer (see Figure 4). The linear and quadratic coefficients were not strongly correlated (see Table 4). Figure 4A shows a scatter plot of the linear and quadratic coefficients for Participant P6 over 41 trials. The crosshair in the center of the scatter plot shows the mean $( \pm 2 S E)$. The crosshairs in Figure 4B show corresponding mean values ( $\pm 2$ SE) for all 10 participants in Experiment 1. Trajectories were analyzed in terms of the quadratic and linear coefficients from the polynomial fits. For example, a trajectory with a nonzero linear term and a zero quadratic term would correspond to a straight-line trajectory at a fixed heading away from the target path, but a trajectory with a zero linear term and a nonzero quadratic term would correspond to a portion of a circular path, with the tangent line pointing straight ahead at the starting point. An example of a typical trajectory is shown in Figure 1. It was fit by a curve with a positive linear coefficient (coefficient $=0.1405$ ) and a negative quadratic coefficient (coefficient $=-0.0322$ ).

## Discussion

The findings in Experiment 1 reveal that people without vision veer significantly when trying to walk a straight line. Furthermore, the results reveal that groups (blind vs. blindfolded) do not differ

Table 3
Mean Veering Error at 9.14 m

|  | Mean error (m) |  |  |
| :---: | ---: | :---: | :---: |
| Participant | Signed | Unsigned |  |
| Blind |  |  |  |
| P1 | 0.30 | 0.77 |  |
| P2 | -0.35 | 0.63 |  |
| P3 | 1.74 | 1.74 |  |
| P4 | 0.61 | 0.71 |  |
| P5 | -0.49 | 0.70 |  |
| Sighted | -0.67 | 0.95 |  |
| P6 | 0.24 | 0.45 |  |
| P7 | 0.45 | 0.84 |  |
| P8 | 0.54 | 0.73 |  |
| P9 | 0.37 | 0.69 |  |
| P10 |  |  |  |



Figure 2. Walking performance without vision. A: Sighted Participant P6's performance on 41 trials. The fan shape created over many trials is typical of this and earlier studies of veering associated with walking without vision. B: Average deviation for each participant, 9.14 m from the origin (see Table 3 for values). In this study, leftward veering is associated with a negative sign, and rightward veering is associated with a positive sign.

## Experiment 2

## Objective

The lack of difference in veering behavior between sighted and blind participants suggests that a history of visual experience is not critical to performance on this task. It remains possible, however, that both sighted and blind people are able to regulate nonvisual walking using feedback from vestibular, proprioceptive, or kinesthetic signals. For instance, the semicircular canals in the inner ear might detect subtle angular accelerations, a cue to detection of curvature of the walking path. Additionally, feedback signals from muscles or joints in the knees or ankles might provide cues to deviation from a straight line. In either case, compensatory motor commands on the basis of these feedback signals contain random (and possibly systematic) errors that could produce deviation from a straight line. If the ability to walk a straight line is limited by errors in perceptual cues about curvature, we would predict that participants who are good at detecting path curvature (i.e., those with a low perceptual error rate) also would be good at walking straight lines. Conversely, those who are poor at detecting curved walking paths should exhibit more severe veering. Given these possibilities, we asked how the extent of veering relates to sensitivity in the detection of curved pathways. We tested the hypothesized relationship between path curvature detection and veering.


Figure 3. Veering consistency across days. This figure shows the ratio of signed endpoint variance $\delta_{\text {total }}^{2}$ across days to the average of the within-day variances $\delta_{i}^{2}$.
A.

B.

10 Participants' Mean Coefficients $\pm 2$ SE


Figure 4. Quadratic and linear contributions to walking trials. A: Distribution of linear and quadratic coefficients over 41 trials for Participant P6. Each data point represents a polynomial fit for a single trajectory. The crosshair in the middle of Panel A shows the mean $\pm 2$ standard error. B: Mean polynomial coefficients for all 10 participants.

## Method

Participants. The same 10 participants from Experiment 1 were tested in Experiment 2.

Testing arrangements and procedure. The two sets of measurements (straight-line walking in Experiment 1 and curvature detection in Experiment 2) were conducted in the same sessions over 3-4 days within a several-week period. Curvature detection trials were evenly spread over all sessions.

We considered several methods for guiding blind participants along curved pathways to minimize rotational cues from the apparatus itself. It was decided that a smooth-operating and nonrigid guiding apparatus would perform best at minimizing confounding feedback. The participants were guided along circular curved paths by a handheld tethered ball, which was attached to a three-wheeled, ballasted cart (see Figure 5). This method was chosen over several other designs because it provided the least amount of rotational torque (a cue for curvature) while still providing an acceptable level of adherence to the intended pathway.
The 10 curves were 4.57 m long and had radii of $9.14 \mathrm{~m}, 12.19 \mathrm{~m}, 18.29$ $\mathrm{m}, 27.43 \mathrm{~m}$, and 36.58 m , left and right. Quantifying curvature by radius is equivalent to saying that the curved path is an arc of a circle of the given radius. For instance, a $4.57-\mathrm{m}$ curve with a radius of 12.19 m is equivalent to a $4.57-\mathrm{m}$ portion of the circumference of a circle with a radius of 12.19

Table 4
Correlations Between Linear and Quadratic Polynomial Coefficients

| Participant | $r^{2}$ | $p$ |
| :---: | :---: | :---: |
| Blind |  |  |
| P1 | .014 | .44 |
| P2 | .003 | .74 |
| P3 | .023 | .32 |
| P4 | .025 | .33 |
| P5 | .001 | .83 |
| Sighted | .009 | .55 |
| P6 | .042 | .20 |
| P7 | .003 | .72 |
| P8 | .002 | .79 |
| P9 | .0001 | .96 |
| P10 |  |  |

m. Participants also were tested on $9.14-\mathrm{m}$ curves, but these data are not reported here.

Each participant performed 194-200 trials over the 3- to 4-day testing period. Some participants were exceptionally good or poor at the curvature detection experiment on Day 1. For these people, the best two curves (near perfect) or worst two curves (near chance) were eliminated from subsequent trials, leaving time for more trials in the relevant range of sensitivity.

After walking along each path, participants were asked to give a twoalternative forced choice (AFC) response for the direction of curvature. A block of trials consisted of 10 or more trials on all 10 different arcs in random order.

We constructed psychometric functions of percentage correct versus radius of curvature. Cumulative Gaussian curves were fit to the data through use of a constrained maximum-likelihood method (see Figure 6A), and threshold was defined as the curvature yielding $90 \%$ correct. The psychometric function took all trials into account during the fitting process.

## Results

The mean threshold in the curvature discrimination task across the 10 participants was a radius of 24.47 m and did not differ significantly between blind participants and blindfolded sighted participants, $t(8)=0.26, p=.60$. Figure 6 B shows that the thresholds ranged from radii of 11.46 m to 36.48 m .

We tested the hypothesis that curvature detection is related to veering behavior by computing the correlations across participants between the curvature thresholds and four measures of veering behavior: mean signed error (offset at 9.14 m ), mean unsigned error (offset at 9.14 m ), mean linear coefficient, and mean quadratic coefficient (see Experiment 1 for details of these measures). We found no significant correlation between curvature thresholds and any of the following four measures of veer: (a) curvature threshold versus signed error, $r(8)=.10, p=.78$; (b) threshold versus unsigned error, $r(8)=.30, p=.39$; (c) threshold versus linear coefficient, $r(8)=-.15, p=.68$; and (d) threshold versus quadratic coefficient, $r(8)=.27, p=.46$.

## Discussion

The results of the present experiment do not support the hypothesis that people who veer the least by the previously mentioned


Figure 5. Curvature detection guiding apparatus.
measures are most sensitive to path curvature. For emphasis of this point, the participant (P3) who was best at curvature detection (threshold radius $=36.48 \mathrm{~m}$ ) also was the one who veered the most in the straight-line-walking task (mean unsigned deviation $=$ 1.73 m ). However, we show in the Model section that a more adequate measure of veering behavior (i.e., variability in stepping) is, in fact, correlated with curvature detection performance.

Cratty (1965) also measured curvature detection of blind walkers. It is difficult to make a quantitative comparison with Cratty's (1965) findings because he used a different method for guiding participants along curved paths (by means of curved guide rails) and a different psychophysical paradigm (i.e., Cratty [1965] used a three-AFC design vs. our two-AFC design). Nevertheless, it appears that participants in our experiment were better at curvature detection. Cratty (1965) stated that participants in his experiment averaged $55.45 \%$ correct (chance $=33.3 \%$ ) on $12.8-\mathrm{m}$ radius
curves. According to his table (see Cratty [1965], Table II, p. 53), participants reached $90 \%$ accuracy when the curved paths had radii of only 5.49 m . Our findings, as noted above, show mean thresholds to be about 24.47 m at $90 \%$ correct (chance $=50 \%$ ). This discrepancy may be due to the difference in psychophysical paradigm and/or the difference in the guiding apparatus. The guiding apparatus used by Cratty (1965) consisted of pairs of curved rails with handheld batons that participants used to touch the rails.

In the curvature detection trials, we tried to approximate the walking speeds that were observed in the veering trials of Experiment 1. Although the guiding apparatus did not generate precisely constant speed, we argue that small speed fluctuations would not influence performance in curvature detection or its relationship with veering performance. We do not believe that the speed fluctuations would be any greater than in Experiment 1 because the ballast in the cart provided smooth velocities and accelerations


Figure 6. Curvature detection performance and threshold calculations. A: Curve detection performance of 1 participant. B: $90 \%$ correct threshold radii for all 10 participants.
throughout each trial. In other words, the inertia of the apparatus should have reduced the fluctuations in speed somewhat over normal walking.

The results of this experiment do not show a direct relationship between sensitivity for detecting curved paths and the ability to walk a straight line. It remains possible, however, that subtler measures of vestibular, kinesthetic, or proprioceptive function might yet reveal such a linkage. Our results prompted us to consider the possibility that undetected noise in the motor system might account for veering behavior. We return to the matter of curvature detection in detail in Appendix B, after introducing and discussing the model.

Before turning to explanations for veering on the basis of the motor system, we considered the possibility that our method for conveying heading to participants (Experiment 1) might have resulted in initial directional uncertainty. On the basis of this possibility, we asked whether providing a perceptual pointer would improve performance in straight-line walking (Experiment 3).

## Experiment 3

## Objective

In Experiment 1, participants were instructed to walk a straight line perpendicular to the edge of a table behind them (physical alignment starting condition). Uncertainty in identifying the meaning of "perpendicular to the table edge" or in translating this direction into a vector for walking might have contributed to orientation uncertainty. In other words, it is possible that we provided our participants with insufficient perceptual input regarding the intended direction for straight-line walking. We asked whether we could reduce veering behavior by providing perceptual pointers, which are explicit indicators of the intended direction.

There is practical value in determining what starting conditions minimize veer. Optimal cues could be implemented in the design of crossing guides at intersections for assisting visually impaired pedestrians. More generally, improved strategies for designating a walking direction could be incorporated into orientation and mobility training techniques.

## Method

Participants. Participants were 5 undergraduate female college students who were normally sighted, ranging in age from late teens to early twenties. All participants were blindfolded. None had participated in Experiments 1 or 2 .

Testing arrangements and procedure. Three counterbalanced starting conditions were tested, including physical alignment, static perceptual pointer, and dynamic perceptual pointer. In each condition, participants performed 39-41 trials over 2 separate days. In the physical alignment condition (a replication of Experiment 1), participants were initially positioned with their backs aligned to a rigid boundary (the edge of a table) perpendicular to the desired walking direction. By squaring their bodies against this boundary, they naturally assumed the appropriate initial orientation. In this task, there was no explicit pointer for the intended walking direction.

In the two perceptual pointer conditions, the desired walking direction was indicated explicitly by aligning the rigid boundary (the edge of the table) parallel to the desired direction of travel. Participants felt the parallel edge and used this cue to determine the appropriate walking direction. The
static perceptual pointer condition relied on a perceptual estimate of the correct starting direction while the participant was stationary at the beginning of the trial. In the dynamic perceptual pointer condition, participants again used the edge of the table, aligned parallel to the desired direction. However, they were able to walk for 1.83 m at the beginning of the trial while continuously feeling the edge of the table before "launching" in the desired direction of travel. In both perceptual pointer conditions, we offset the table by 0.30 m to position the participant's center of gravity over the desired walking pathway. We reasoned that the perceptual alignment methods should provide enhanced perceptual cues (either static or dynamic) for the desired walking direction. To the extent that these perceptual cues provide improved information about orientation, we would expect better performance (less veering). If perceptual factors do not limit performance in the veering task, the physical alignment condition might yield equivalent, or perhaps even better, performance than would the conditions with the additional perceptual cues.

## Results

Veering patterns in the physical alignment condition were similar to the data collected with the same procedures in Experiment 1. Figure 7 and Table 5 show an analysis of three dependent variables (endpoint, linear coefficient, and quadratic coefficient) on three different starting conditions. Additionally, the effects on initial orientation and stepping biases (which are related to the linear and quadratic components, respectively) are shown. We discuss initial orientation and stepping bias in detail as model parameters in the Model section. These measures are highly correlated with the linear and quadratic coefficients, respectively. Initial orientation can be thought of as the initial direction that the walker assumed at the beginning of each trial. Stepping bias can be thought of as the mean tendency to change heading at each step.

Figure 7 shows that the addition of the perceptual cues did not reduce veering compared with the physical alignment condition. Furthermore, the linear coefficient was driving the effect of endpoint. This observation suggests that initial orientation was being affected by the initial cues (or position) rather than by the stepping bias (as shown in the relatively small effect on the quadratic component).

It is important to note the possibility of an effect of the lead foot, that is, the foot that took the first step in each trial. We tested whether the lead foot had an effect on endpoint or on linear and quadratic components of trials. In the physical alignment condition, an ANOVA revealed an effect of lead foot on the linear component. Leading with the right foot resulted in an average linear slope of -0.0383 , whereas leading with the left foot resulted in an average linear slope of $0.0160, F(1,193)=6.90, p=.033$. We found no effect on the endpoint, $F(1,193)=1.05, p=.357$, or on the quadratic component, $F(1,193)=.111, p=.755$. The absence of an effect on the quadratic component suggests that there is no lead foot effect on the stepping bias estimation. There may be practical implications on the effect of lead foot for orientation and mobility training, that is, the lead foot probably influences the walker's initial orientation.

Figure 7A shows that the endpoint offsets were similar for the physical alignment condition and for the dynamic perceptual pointer condition, but the offsets were greater (i.e., more veer) for the static perceptual pointer condition. Detailed analysis of the polynomial fits to the trajectories in the three conditions revealed that both the linear and quadratic coefficients were significantly


Figure 7. Effects of three starting conditions on five measures of veering trajectories. All data were analyzed in absolute values. Panel A shows box plots of the distributions of endpoint offsets at 9.14 m . Panels B and E show the effect of starting condition on linear and quadratic coefficients of the trajectories. Panels D and G show the effect of starting condition on initial orientation and step direction bias. Panels C and F show the linear relationships between these measures and the polynomial coefficients (see Table 5 for statistics). Coef $=$ coefficient; Stat $=$ static; Dyn $=$ dynamic; Phys $=$ physical; Init Orient $=$ initial orientation; deg $=$ degrees.
larger in the static perceptual pointer condition than in the other two conditions but that the effects on linear coefficients were substantially more pronounced than were the effects on quadratic coefficients. A repeated-measures ANOVA showed that $54 \%$ of the variation in linear coefficients can be accounted for by the variation in the three starting conditions (see Figure 7B), whereas a repeated-measures ANOVA on the quadratic coefficients showed an effect size of only $13.4 \%$ (see Figure 7E; see Table 5 for statistics).

## Discussion

An explicit indicator of intended walking direction did not reduce veering behavior. In fact, in the case of the static perceptual pointer condition, veering actually increased compared with the physical alignment condition, in which there was no explicit pointer. The increased veer is due mainly to a larger linear com-

Table 5
Effects of Three Starting Conditions on Five Measures of Veering Trajectories

| Measure | $d f$ | $F$ | $\eta^{2}$ | $R^{2}$ |
| :--- | :--- | :--- | :--- | :--- |
| Effect |  |  |  |  |
| End point | 2 | $150.52^{* * *}$ | .436 |  |
| Linear coefficient | 2 | $228.65^{* * *}$ | .540 |  |
| Initial orientation | 2 | $261.46^{* * *}$ | .573 |  |
| Quadratic coefficient | 2 | $30.06^{* * *}$ | .134 |  |
| Stepping bias | 2 | $18.75^{* * *}$ | .088 |  |
| Correlation |  |  |  |  |
| Initial orientation versus linear coefficient |  |  | .960 |  |
| Stepping bias versus quadratic coefficient |  | .914 |  |  |
| ${ }^{* * *} p<.001$. |  |  |  |  |

ponent in the participants' trajectories. This increase is likely accounted for by larger errors in initial orientation at the start of the trajectories. In the Model section that follows, we use the variable representing initial orientation (shown in Figure 7D). The lack of improvement in straight-line walking when an explicit cue for walking direction is present implies that veering is not due to insufficient information about the intended walking direction.

Performance was better in the dynamic pointer condition than in the static pointer condition and was equivalent to performance in the physical alignment condition. The dynamic condition allowed the participant to get a moving start by following the edge of the table. We speculate that the benefit occurred because the moving start and associated inertia provided alignment at the start of the trajectory, equivalent in accuracy to the physical alignment condition.

These findings imply that physical pointers may not be effective in designating walking direction for visually impaired people (e.g., in crossing a street). A better strategy may be providing a guide bar perpendicular to the crossing direction and encouraging the pedestrian to back against it to establish the starting orientation. Alternatively, if a guide rail is aligned parallel with the crossing direction, the visually impaired pedestrian could follow the rail to get a moving start before crossing the street.

The linkage between linear coefficients and orientation at the beginning of trials is evident from this experiment. In the Model section, we discuss initial orientation and an additional measure of veering that captures the average error in stepping direction. These measures are closely linked, conceptually, to the linear and quadratic coefficients of the polynomial fits and are closely correlated with them (see Figure 7C, Figure 7F, and Table 5).

## Model

## Objective

The results of our curvature detection experiment (Experiment 2) imply that something other than sensory feedback may limit performance on straight-line walking. Moreover, perceptual pointers provide a less reliable cue to initial orientation than does physical alignment (Experiment 3) and, hence, provide greater initial orientation errors. These observations raise the possibility that veering may be due to noise in the motor system that is not detectable by vestibular or kinesthetic feedback. We hypothesized that small amounts of independent motor noise (occurring in each individual step) can explain veering behavior when participants attempt to walk a straight line without vision. To explore this idea, we developed a random-walk model to simulate noisy steps in simulated walking trials.

## Method

The model presented here simulates the effects of noisy steps (variability in the length and direction of individual steps) on the immediate position and orientation of a walker. We intended the model to be a radical simplification of actual walking in order to focus on the causes of veering. In particular, the model considers only the two-dimensional (2D) position and direction of steps, intentionally ignoring biomechanical aspects of walking (including the distinction between left and right feet as well as deviations in step height).

The walking model, summarized in Figure 8, contains six parameters: mean initial orientation prior to the first step $\theta_{\text {init }}$, initial orientation
variability (occurring over numerous trials) $\sigma_{\theta}^{2}$, mean step length $l$, step length variability $\sigma_{l}^{2}$, mean step direction bias $\beta$, and step direction variability $\sigma_{\phi}^{2}$. The initial orientation variable simulates the initial direction of travel and has a value of 0 if the walker is initially aligned with the desired heading. Mean step length refers to the average distance traveled in a single step (in this article, step length is equal to 0.5 stride length). Step direction bias $\beta$ represents a bias in the step direction away from orientation in the previous state. A bias of $+1^{\circ}$, for example, refers to a walker whose average step rotates $1^{\circ}$ clockwise from the local straight-ahead orientation. Accumulated step direction bias across a path results in consistent deviations from a straight line. Step length variability and bias variability are the step length variance and step direction variance, respectively. Each of the parameters in our model can be estimated from the human trajectory data as follows.

Average step length $l$. We estimated average step length by dividing the distance traveled $\left(d_{k}\right)$ on each trial $(k)$ by the number of steps $\left(N_{k}\right)$ recorded from the video data in Experiment 3: $l=\frac{1}{K_{k=1}^{K}} \sum_{k}^{K} \frac{d_{k}}{N_{k}}=\frac{1}{K_{k=1}^{K}} \sum_{k}^{K} l_{k}$.

Step length variance $\sigma_{l}^{2}$. An indirect measure of variability was computed from the variance of $l_{k}$ across trials, scaled up by the average number of steps per trial. This estimate was justified by the observation that (assuming independent steps are taken) the standard deviation of $l_{k}$ would be reduced over the variance of individual steps by a factor of $\sqrt{N}=\sqrt{\frac{1}{K} \sum_{k=1}^{K} N_{k}}$, on average. Essentially, we converted an estimate of the standard error of the mean into a standard deviation.

We estimated the remaining four parameters from the polynomial fits to the human data by resampling individual steps along the fitted trajectories. In particular, for each polynomial fit, the cumulative arc length, $s(x)$, was computed from the polynomial $y(x), s(x)=\int_{0}^{x} \sqrt{\left(\frac{d y}{d x}\right)^{2}+1} d x$.

Because $s(x)$ is monotonic in $x$ and represents the cumulative distance along the path, the $x$ values corresponding to equally spaced steps of length $l$ along the path were computed from $s(x)^{-1}$. The $y$ values corresponding to these $x$ values were then computed from the fitted polynomial. The result was a series of step vectors $\left(\mathbf{u}_{1}, \ldots, \mathbf{u}_{M}\right)$ that divide the trajectory into $\mathbf{M}$ pieces of length $l$. The orientation of each step vector $\mathbf{u}_{j}$ with respect to the $y$ axis was computed simply as $\psi=\tan ^{-1}\left(\mathbf{u}_{x} / \mathbf{u}_{y}\right)$.

Given the step vectors, the remaining parameter estimates are easily described. To preserve information about individual variation, we computed all estimates within participants and within days.

Initial orientation. We estimated the initial orientation $\theta_{\text {init }}$ by averaging the orientation of the first step vector across trajectories for each participant: $\theta_{\text {init }}=\frac{1}{K} \sum_{k=1}^{K} \psi_{1}^{k}$, where $K$ indexes the trajectory number. For simulation purposes, the variance of $\psi_{1}$ was computed, as well.

Step direction bias $\beta$. We estimated the step direction bias $\beta$ by averaging the orientation difference between steps both within and across trajectories: $\beta=\frac{1}{K(N-1)} \sum_{k=1}^{K} \sum_{j=2}^{N_{k}}\left(\psi_{j}^{k}-\psi_{j-1}^{k}\right) \approx \frac{1}{K_{k=1}^{K}} \sum_{k} \beta_{k}$, where $K$ indexes the trajectory number, $N$ represents the average number of steps per trial, and $j$ indexes the step number within a trajectory.

Step direction variance $\sigma_{\phi}^{2}$. An accurate measure of the step direction variance $\sigma_{\phi}^{2}$ is subtler. Because the steps were derived from polynomial fits that smooth the step variability within a trajectory, the variability of $\psi_{j}^{k}$ across steps $j$ is smaller than the underlying step variability. To get around this difficulty, we used an argument similar to that used for step length variability, appropriately scaling the variance of the average step direction across trials: $\sigma_{\phi}^{2}=\frac{N-1}{K} \sum_{k=1}^{K}\left\{\beta_{k}-\beta\right\}^{2}$, where $K$ indexes the trajectory number. Our human data suggest that angular stepping variability does not change much across individuals or days.

# Model Parameters 

Orientation change


Observer Position at step $k$

$$
\mathbf{P}_{k}=\left[\begin{array}{l}
x_{k} \\
y_{k}
\end{array}\right]=\left[\begin{array}{l}
x_{k-1} \\
y_{k-1}
\end{array}\right]+\left[\begin{array}{l}
v_{k}^{x} \\
v_{k}^{y}
\end{array}\right]
$$

Observer Orientation

$$
\begin{gathered}
\theta_{k}=\phi_{k}+\theta_{k-1} \\
\theta_{1}=\theta_{\text {init }}
\end{gathered}
$$

| Step Model | Bias | Step Length |
| :---: | :--- | :---: |
| $\mathbf{V}_{k}=\left[\begin{array}{c}r_{k} \cos \left(\phi+\theta_{k-1}\right) \\ r_{k} \sin \left(\phi_{k}+\theta_{k-1}\right)\end{array}\right]$ | $\phi_{k}=\beta+\varepsilon_{\phi}$ | $r_{k}=l+\varepsilon_{l}$ |

Figure 8. Stepping model for walking. The walker's state is specified by his or her global two-dimensional position $\mathbf{P}_{k}$ and orientation $\theta_{k}$. Each step of average length $l$ is in the direction of the walker's current orientation, but the executed step $\mathbf{V}_{k}$ differs by the addition of noise in both the length $\left(M=l\right.$, variance $\left.=\sigma_{l}^{2}\right)$ and direction of the step $\left(M=\beta\right.$, variance $\left.=\sigma_{\varphi}^{2}\right)$. By the end of each step, the walker's orientation has changed to point in the direction of the executed step. Initial orientation is a random variable across walks.

We used these parameters to compute a noisy step $\mathbf{v}_{k}$, which modifies the walker's position and orientation as follows: The first step of the simulated walk is in the direction of the initial orientation value. At the end of each subsequent step $k$, the new heading $\theta_{k}$ is perturbed away from its previous value $\theta_{k-1}$ by the current step. Accordingly, heading varies from step to step in a kind of random wobble. However, for small values of the step direction variance, the trajectories remain smooth.

Parameter estimates for individual performance on each day (Experiments 1 and 3 on veering) were used in the model (see Tables 2 and 6). From the physical alignment condition in Experiment 3, we were able to obtain estimates for initial orientation, bias, and step length parameters, whereas in Experiment 1, we were able to obtain estimates for initial orientation and bias, assuming an average step length of about 0.62 m (estimate from Experiment 3 data). From these data, models for each person/day combination were evaluated on Experiment 3 participants. The parameterized models have three free parameters-mean orientation, mean step length, and mean bias-and three fixed variance parameters that are based on population estimates.

Given the parameter estimates from the last section, it is straightforward for us to simulate walking paths by means of the model. We generated each model trajectory using the following three steps: (a) We generated an initial orientation by sampling from a Gaussian distribution whose mean and variance were derived as noted in the Initial orientation subsection of the Model section; (b) we generated the step length and bias at each step by sampling from the corresponding Gaussian distributions, and (c) we updated the position and orientation state variables according to the equations given in Figure 8.

To make the comparison with human data fair, we treated the simulated trajectories exactly the same way as we did the human data. In particular, each simulated trajectory was sampled every 1.52 m , after which it was analyzed exactly like the human data. Using the same techniques implemented in human experiments, we fit individual modeled trajectories with second-order polynomials by least squares.

Matlab code for implementing the model is detailed in Appendix A.

## Results

We first parameterized the model using pooled data from our human experiments (see marginal means in Table 6 for mean estimates). This yielded a simulated walker whose trajectories and ensemble statistics were similar to those of average walkers (see Figure 9). The fan-shaped distribution of modeled trajectories is similar to the corresponding cluster of human trajectories and is due to the stochastic variables, particularly the noise in stepping direction $\sigma_{\phi}^{2}$.

In the simulation shown in Figure 9, Panels A-C, we used the following fixed parameters (based on Experiment 3 results; see Table 6): $\theta_{\text {init }}=-1.15^{\circ}, \sigma_{\theta}^{2}=2.18^{\circ}, l=0.62 \mathrm{~m}, \sigma_{l}^{2}=0.10 \mathrm{~m}, \beta=$ $-0.12^{\circ}$, and $\sigma_{\phi}^{2}=1.33^{\circ}$. Incidentally, we conducted simulations showing that step length variability $\sigma_{l}^{2}$ (at any reasonable magnitude) does not have a significant effect on the veering behavior of our model. Mean step length $l$ has a scaling effect in the model. For

Table 6
Walking Parameter Estimates (Experiment 3)

| Participant | Day | Initial orientation $\left({ }^{\circ}\right)$ |  | Step length (m) |  | Bias ( ${ }^{\circ}$ ) |  | No. trials | Mean steps (n) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $M$ | $S D$ | M | $S D$ | $M$ | $S D$ |  |  |
| Participant number |  |  |  |  |  |  |  |  |  |
| P11 | 1 | 0.47 | 1.79 | 0.59 | 0.085 | 0.39 | 1.16 | 20 | 15.7 |
|  | 2 | -1.54 | 1.98 | 0.58 | 0.081 | -0.19 | 1.38 | 21 | 15.8 |
| P12 | 1 | 2.11 | 3.70 | 0.61 | 0.14 | -0.10 | 1.49 | 21 | 15.0 |
|  | 2 | 0.85 | 2.20 | 0.63 | 0.083 | -0.43 | 1.12 | 20 | 14.5 |
| P13 | 1 | 0.37 | 1.97 | 0.57 | 0.055 | -0.17 | 0.81 | 20 | 16.2 |
|  | 2 | -3.47 | 1.79 | 0.58 | 0.034 | -0.58 | 0.83 | 20 | 16.1 |
| P14 | 1 | -2.68 | 2.62 | 0.51 | 0.27 | -0.48 | 1.58 | 19 | 18.5 |
|  | 2 | -0.63 | 1.68 | 0.62 | 0.073 | -0.38 | 1.56 | 20 | 14.8 |
| P15 | 1 | -2.63 | 1.62 | 0.71 | 0.087 | 0.36 | 1.45 | 20 | 13.0 |
|  | 2 | -4.36 | 2.48 | 0.78 | 0.088 | 0.35 | 1.94 | 20 | 11.9 |
| Marginal means |  | -1.15 | 2.18 | 0.62 | 0.100 | -0.12 | 1.33 |  |  |
| Other studies |  |  |  |  |  |  |  |  |  |
| Bauby \& Kuo (2000) ${ }^{\text {a }}$ |  |  | 1-3 | 0.76 | 0.119 |  | 1-3 |  |  |
| Donelan, Shipman, Kram, \& Kuo $(2004)^{a}$ |  |  |  | 0.72 | 0.091 |  | 1-2 |  |  |

${ }^{\text {a }}$ Means and standard deviations for initial orientation and bias were not provided by these authors; thus, $M$ and $S D$ columns are left blank.
example, larger steps produce fewer steps for a given distance and, hence, less overall veering, whereas smaller steps have the opposite effect. The simulated trajectories were analyzed in the same way as were human trajectories: Sampling was done at the same $1.52-\mathrm{m}$ intervals, and polynomials were fit to the data (see Figure 1). Figure 9A shows sampling and polynomial fits for the averaged model (Figure 9D shows Participant P2). Figure 9B shows the corresponding coefficient scatter plot for the averaged model (Figure 9E shows Participant P2).

If individual model trajectories behave like human trajectories, then we would predict that the distribution of model polynomial coefficients would match the distribution of coefficients found in human data. In Figure 9, we can compare human coefficients (see Figure 9E) and modeled coefficients (see Figure 9B). Recall the simulated trials shown in Figure 9A. The trajectories of the model produced coefficients that were uncorrelated, similar to coefficients of blind and blindfolded human walkers.

A further comparison of the modeled trajectories can be made by a plot of the residuals between data and least-squares polynomial fits (see Figure 9C and 9F). This similarity shows that the polynomial fits are equally good for the model and human trajectories.

Although the average model is an idealization of actual walking, we believe that it captures the aspects of walking most important for an explanation of veering. As illustrated in Figure 9, individual trajectories and their accompanying polynomial coefficients are indistinguishable from the performance of a typical human participant.

Our findings in Experiments 1 and 3, confirming earlier findings by Guth and LaDuke (1995), show that individuals behave slightly differently from day to day and from one to another. These individual differences are not accounted for by the average walker
model. To account for these differences, we derived parameter estimates from individuals (see Tables 2 and 6) and ran simulated trials on the basis of those parameters. Next, we show that the model can replicate individual and day-to-day variations in human performance through day-to-day variations in the means of initial orientation, step length, and step bias.

Data collection in Experiment 3 included counts of the number of steps in each human trajectory. The number of steps for each trajectory was translated into average step length and is listed in Table 6. Using parameter estimates for individuals from Experiment 3 (see Table 6), we simulated 100 modeled trajectories for each of the 10 individual-day combinations. Linear and quadratic coefficients of simulated trials were compared with corresponding values from each set of human trajectories. We used Bonferroniadjusted Hotelling's $t^{2}$ tests to compare the linear-quadratic component combinations between human estimates and modeled estimates. The Hotelling $t^{2}$ test is essentially an $n$-dimensional $t$ test that researchers can use to take both components of polynomial coefficients into account at once. With these parameter estimates, none of the simulated data were significantly different from corresponding human data. Each simulation has three free (individually estimated) parameters, including initial orientation, bias, and step size. Patterns of the model and corresponding human performance also are visually similar, as noted in Figure 10.

We constructed a model so that we could investigate how well model trajectories match human veering trajectories. For the model to be compelling, two conditions should be present: (a) it should produce individual trajectories that look like human trajectories, and (b) the statistical properties of an ensemble of model trajectories should match those of human trajectories. Assessment of the similarities of simulated trials showed that polynomial fits to


Figure 9. Human and model comparison. Panel A shows modeled data with fitted curves, Panel B shows resulting polynomial coefficients, and Panel C shows their corresponding residuals for modeled trajectories on the basis of parameters from an average walker (see marginal means in Table 6). Empirical data from Participant P2 are shown in Panels D, E, and F.
modeled trajectories matched those of participants in both the value of coefficients and the residuals to the fits.

## Discussion

Although the model captures salient aspects of veering behavior, it involves a number of abstractions away from actual walking behavior. For example, the walker's position in the model represents the projection of some distinguished point (such as the center of mass) onto the ground plane (this can be seen similarly with orientation). A more complete characterization would involve the position of several points (e.g., head location, center of mass, center of hips) in three-dimensional (3D) position. Nevertheless, we believe that our model parameters can be identified with biomechanical properties of real walkers. Initial orientation is simply the initial walking direction (where walking direction is defined by the direction of motion of the participant's center of mass), and initial orientation variability is the variation in initial walking direction between trials. The model steps, however, involve more abstraction.

Model steps do not directly represent where the feet are placed. Instead, they represent the consequences of foot placement and postural instability during a step on the walker's position and direction, rather than the foot placement per se. Actual steps are quite complicated, and researchers choose foot placement controls to simultaneously satisfy two distinct goals: transporting the walker and maintaining postural stability (Bauby \& Kuo, 2000; Donelan, Shipman, Kram, \& Kuo, 2004). In our model, step direction represents the walking direction at each step, and step length represents the distance traveled in that direction. Because steps are in the forward direction, on average, the step length parameter should represent actual forward step length (distance between steps in the walking direction). Relating step direction to actual steps requires a discussion of how postural stability is maintained by step placement.

The simplest model of 3D walking dynamics (Garcia, Chatterjee, Ruina, \& Coleman, 1998; McGeer, 1990) consists of two cylindrical legs connected by a pelvis, with ball joints at the hips, and curved feet connected by pin joints to the end of the legs.


Figure 10. Real and simulated trajectories. Each panel shows a summary for trials on the basis of the estimated parameters from Experiment 3. Solid curves show the means ( $\pm 1 S E$ ) of Experiment 3 trajectories. Simulated trajectories are shown with three dotted curves representing the mean $( \pm 1 S D)$. Sims $=$ simulations; Init $=$ initial orientation.

Surprisingly, when this model walker is placed on a tilted platform, it is passively stable in the walking direction, which means that it requires no energy expenditure or control signals for maintenance of both walking gait and postural stability in the walking direction (Garcia et al., 1998; Kuo, 1999, 2001; McGeer, 1990). However, this passive stability does not extend to the direction that is lateral to the walking direction (Bauby \& Kuo, 2000; Donelan et al., 2004; Garcia et al., 1998). To maintain balance, walkers must actively control lateral postural stability, which they accomplish, in large part, by varying the lateral placement of foot position
(Bauby \& Kuo, 2000; Donelan et al., 2004). Because the center of mass must be brought sufficiently over the foot during the next step, lateral foot placement results in both positional noise in the lateral direction and (more important) changes in the orientation of the walker. Changes in direction should occur from torques generated by foot impact, and these changes should reduce twist at the ankle. Foot impact events are the most important part of the walking cycle because nearly all force control is applied at impact (Donelan et al., 2004). At impact, contact forces generated by a foot placement forward and to the side produce a force component
in the desired lateral direction but also produce a torque component around the walker's main axis (the gravity axis) in the direction of the step that modifies the walker's direction unless accurately sensed and corrected. Thus, lateral foot placements should produce perturbations in the direction of the walker; we modeled these perturbations with our step direction parameter. In particular, systematic direction perturbations from lateral foot placement can be associated with mean step direction bias, whereas random perturbations would be associated with step direction variance.

Given the previous discussion, we can try to relate the values for the parameters in our model to results from previous studies. In particular, we focus our attention on two recent studies (Bauby \& Kuo, 2000; Donelan et al., 2004) that measure step length and step variability in the forward and lateral directions. The values reported in these two studies are converted into the units of measurement used in this article and are summarized at the bottom of Table 6. Step length and step variability comparisons were straightforward unit conversions. In general, step length is about 0.8 of the walker's leg length, but step length is difficult to directly compare across studies because the walking speed used in our study was probably slower than that used in the comparison studies (because of the nature of nonvisual walking vs. sighted walking). We further discuss the effects of step length and speed on veering below. We converted lateral foot placement variability into step direction variability by making the simplistic assumption that a lateral foot placement of $x$ degrees to the side of the current walking direction would result in a corresponding change in direction at the next step. Finally, because initial orientation variability represents the variability in the first step direction, we would expect its value to be nearly the same as that of step direction variance. Given the simplifying assumptions, we believe that the close agreement between our values and those measured in carefully controlled studies using state-of-the-art tracking methods bolster the case for the viability of our model.

Relative role of model parameters on veering. The results of our modeling effort suggest three important and distinct causes of veering from a designated straight line: (a) error in initial direction, captured by the initial orientation and variance parameters; (b) systematic errors in stepping direction, modeled by step direction bias; and (c) undetected random errors in step direction, modeled by step direction variance. The difference between these causes and the role of the other parameters is discussed later in this section.

We included the initial orientation parameters in the model to account for the impact of initial misalignment on veer trajectories. Even if no other bias or source of noise were present, a misalignment by angle $\theta$ at the beginning of a trial results in an offset by a distance $l \cdot \tan \theta$ at the end of a trial from the straight-ahead length $l$. For small angles, the distance would be $l \cdot \tan \theta \approx l \cdot \theta$, with $\theta$ measured in radians. From our estimates in Table 6, the mean absolute initial orientation misalignment of our participants was $1.15^{\circ}$ ( 0.02 radians), producing an average offset for $l=9.14 \mathrm{~m}$ of 0.18 m . Although smaller than the mean absolute offset of 0.82 m that we observed in Experiment 1 (see Table 3), it is clear that the initial orientation error is a contributor to the veering effect. Although errors of initial orientation may be important in real-world situations, such as crossing a street at an intersection nonvisually, this parameter represents
only a starting direction in the model and is not relevant to the stochastic properties of veering.

Many earlier proposals for explaining veering behavior (reviewed by Cratty, 1965, 1971; Guth \& LaDuke, 1994)—proposals such as biomechanical asymmetries and dominance or difference in leg strength or leg length - can be reinterpreted as proposals for sources of systematic errors in step direction. On the basis of an average mean step direction bias of $-0.12^{\circ}$, a step length of 0.62 m , and an average of 15 steps, we can estimate the effect of the stepping bias as follows: Each step introduces a deviation similar to that of the initial orientation but affects only the remaining trajectory (i.e., there is a diminishing effect on overall deviation as each step approaches the end). Thus, the effect of bias is a cumulative sum of the last 14 steps, $\sum_{j=1}^{14} j \cdot s \cdot \tan \theta$, which results in an average effect of 0.13 m , which is small compared with 0.82 m . Thus, although some of our participants show strong systematic veering tendencies, the modeling results suggest that the impact of step direction variance is the dominant factor that influences veering. This observation dovetails with our findings and those of others (Cratty, 1971; Guth \& LaDuke, 1995), which show that the direction of veer is not one sided.

The dominant cause of veering in an established walk appears to be undetected errors in step direction. The unexplained mean offset is 0.50 m , which agrees exactly with the effect of step direction variability observed in our simulations. The idea that veering is caused by undetected motor error constitutes a novel explanation for the veering phenomenon. Note that the term motor error does not distinguish between errors in step planning and execution. Because blind walking must rely heavily on a dead-reckoning type of movement accumulation for assessment of relative position and orientation, the idea that motor error is a problem is no surprise. However, motor error is only a problem to the extent that it is not perceived. For motor error to go unnoticed, the typical amount of motor error should be below perceptual threshold. If the typical amount of motor error is below perceptual threshold, then the walker simply does not know that errors are being introduced at each step. To test this idea, we converted thresholds from the curvature detection experiment (Experiment 2) into an equivalent perceptual error, measured as a variance on orientation change per step through use of a simple formula derived in Appendix B: $\sigma_{\text {perceived }} \geq \frac{89.5}{R}$. A threshold radius of 24.48 m translates into a perceptual error variance of $1.12^{\circ}$, which is within measurement error of the step direction variance of $1.33^{\circ}$. However, because participants actually took 6-8 steps before making a decision (along the $4.57-\mathrm{m}$ curves), the perceptual variance is almost surely higher (for reasons explained in Appendix B). This result is also much larger than the estimated step direction bias values, suggesting that participants are unaware of their systematic walking errors. The similarity between perceptual variance and step direction variance is unlikely to be an accident: If a motor error is detected, it should be corrected. This idea suggests that participants with the highest curvature detection thresholds (i.e., those most sensitive to curve detection) should have the smallest step direction variances. To test this idea, we first counted the mean number of steps measured from Experiment 3 and used this value to estimate stepping variability in participants from Experiment 1.

After we estimated the variances for each participant, we correlated these values with the inverse of each participant's curvature detection threshold measurement (see Appendix B for details). In fact, we found a significant correlation ( $r=.64, p=.045$ ) between perceptual variance (a curvature detection measure) and step direction variance (blind: $r=.72$, sighted: $r=.60$ ). This correlation would likely increase if the exact number of steps in the trajectories from Experiments 1 and 2 were known.

Not surprisingly, no significant relationship was found between (a) perceptual variance and (b) step direction bias or initial orientation. Thus, curvature detection is related to one of the causes of veering in that it acts as an upper bound on step direction variability. A simple explanation can be offered for the discrepancy between these results and the results of Experiment 2 (i.e., there is no direct correlation between curvature detection thresholds and measures of veering), namely, that the correlation between the perceptual and step direction variances does not survive the conversions to curvature detection thresholds and veering measures because of the presence of nonlinear transformations and the fact that step direction variance is only one of several factors that potentially causes veering. In other words, an individual's inability to detect the curvature of the path that he or she is on is not a direct cause of veering.

Intuitively, we would expect the mean step length and steplength noise parameters of our model to be less critical to the veering phenomenon than the other variables. We found this through exploration of the model's behavior. Recall that changes in mean step length have a scaling effect, whereas variability in step length has a negligible effect on veering. But if the model were used for estimation of distance traveled, rather than offsets in direction, the step-length variables would be more important. In a recent empirical study in our lab (Mason, Legge, \& Kallie, 2005), we estimated mean step length and step-length variability for normally sighted and visually impaired walkers. Our interest was motivated by the possibility of using step counts (recorded by a computer-readable pedometer) to estimate distances traveled and then using these estimates to update the pedestrian's position on a computer-readable map. In the study, participants repeatedly walked a fixed distance. We used the number of steps taken to estimate mean step length and variability across trials. We found no significant differences in mean step length or step-length variability between normally sighted and visually impaired walkers (some of whom used a white cane or dog guide). For this group of 18 participants, mean step length at their preferred pace ranged from 0.55 to $0.88 \mathrm{~m}(M=0.74 \mathrm{~m})$. Across the group, the variability was very small, near $2 \%$, meaning that the number of steps the participants took to walk 24.38 m varied by only about $2 \%$ across trials for a given participant. It is likely that researchers could use the model presented in this article, with particular attention to mean step length and variability, to account for variations in distances walked by human pedestrians.

In addition, our model does not take into account walking velocity, which Cicinelli (1989) found was inversely related to veering. We do know, however, that preferred step length increases with walking velocity both empirically (Grieve, 1968; Mason et al., 2005) and theoretically (Kuo, 2001). In our model, an increase in mean step length has the effect of reducing veer because longer steps mean that a given distance is covered in fewer steps, contributing fewer noisy samples to the trajectory. For
example, for our 9.14-m distance, we compared 10,000 simulated trials with step lengths of 0.61 m and 0.91 m , keeping all other parameters equal to zero except for stepping noise ( $\sigma_{\phi}^{2}=1.33^{\circ}$ ); the corresponding standard deviations of the endpoints were 0.50 m and 0.42 m , respectively. This translates to $20 \%$ greater endpoint variability with the shorter steps after 9.14 m of walking. The effect of velocity might, therefore, be related to its effect on mean step length.

Experiments 1 and 3 gave us the opportunity to make parameter estimates of human walking performance, which would then be used in the modeling effort. Using average parameter estimates from Experiment 3, we were able to simulate walking over longer distances. In 1995, Guth and LaDuke measured 3 participants' veering behaviors at 25 m on 3 separate days. They reported three error measurements describing performance: constant error, variable error, and absolute error. Constant error was the mean offset at 25 m , variable error was the standard deviation, and absolute error was the mean of the absolute values. From their Table 1 (Guth \& LaDuke, 1995, p. 31), the constant errors ranged from -4.94 m (left) to 5.75 m (right), variable errors ranged from 1.97 to 3.70 m , and absolute errors ranged from 1.84 to 5.75 m .

Using average parameters in our model, as discussed previously in connection with Figure 9, we ran 10,000 simulated trajectories out to a distance of 25 m and calculated the same three measures of error used by Guth and LaDuke (1995). The simulation produced a constant error of -1.64 m (left), a variable error of 2.43 m , and an absolute error of 2.30 m . These values are all well within the range of Guth and LaDuke's (1995) human participants. The congruence of our model-based extrapolations with the data of Guth and LaDuke (1995) enhance our confidence in the viability of our model and also provide an indirect verification that the veering behavior of our participants was similar to the veering behavior measured by these authors.

In this article, we used the model to account for veering behavior over a small distance of 9.14 m . Over larger distances, the step-by-step accumulation of small errors in direction can result in quite severe misorientation. The model may explain why people sometimes get lost when attempting to walk moderate distances nonvisually.

## General Discussion

Why do people veer away from their intended paths when attempting to walk a straight line in the absence of vision? Veering is an everyday problem for blind mobility and has been the downfall of many normally sighted people who have become lost in blizzards, severe fog, or the dark. Our theoretical answer is that directional errors resulting from noisy steps are a major cause for the veering behavior observed in blind walking. We estimate that individual steps in blind walking have directional variability with a standard deviation of about $1.3^{\circ}$. Accumulation of the effects of errors of this size across steps leads to curved walking trajectories and deviations away from intended pathways. Deviation is modest for short distances such as 9.14 m but is increasingly more severe at greater distances. If, as our results suggest, these small errors in directional stepping are not perceptually detectable in the absence of visual or auditory targets for reference, the walker has no reliable sensory signal for correcting the deviations away from the intended path.

In the Model section, we presented a simple stochastic model of veering behavior. In addition to step direction bias and noise, the model includes variability in step length and initial heading direction. The model comprises three noise parameters, along with the corresponding mean values of the three random Gaussian variables. We used this model to account for the veering behavior of our human participants (Experiments 1 and 3), including day-today and individual variations, and to compare our data with the prior findings of Guth and LaDuke (1995).

In Experiment 1, we measured veering by asking participants to repeatedly walk a $9.14-\mathrm{m}$ straight path. Participants exhibited curved trajectories that were well fit by polynomials with linear and quadratic components. Across the 10 participants, the mean unsigned deviation away from the intended path at the $9.14-\mathrm{m}$ distance ranged from 0.45 m to 1.74 m (see Table 3). In agreement with the findings of Guth and LaDuke (1995), we found significant individual differences and day-to-day differences in veering. In this experiment, we compared the veering behavior of 2 groups of participants: a group of 5 blind participants and a group of 5 blindfolded, sighted participants. We made this comparison to determine whether experience with visual perception would affect veering behavior. There were no significant differences between the 2 groups.

In Experiment 2, we asked whether ability to detect curved walking paths was related to veering behavior. If people monitor perceptual signals from their vestibular organs or from joints or muscles to learn about deviations away from a straight path, we predicted that people who are better at detecting path curvature should veer less. In the experiment, we measured thresholds for path curvature detection by having participants walk along $4.57-\mathrm{m}$ arcs of circles curving left or right, with radii ranging from 9.14 m to 36.58 m . On each trial, a forced-choice decision about whether the path curved left or right was made. Across the 10 participants, threshold radii $(90 \%$ correct criterion) ranged from 11.46 m to $36.48 \mathrm{~m}(M=24.48 \mathrm{~m})$. No significant correlation was observed between these thresholds and any of several measures of veering deviation. From this result, we concluded that veering is not limited by a perceptual capacity to detect curvature away from a straight line.

In Experiment 1, we implicitly defined the intended direction for walking by aligning the participant with his or her back to a boundary perpendicular to the desired direction. In Experiment 3, we asked whether performance could be improved (i.e., veering could be reduced) by providing an explicit perceptual cue for the intended direction. The cue was an elongated pointer (edge of a table) aligned with the intended walking direction. We found that veering actually increased when participants relied on the static perceptual cue and did not use physical alignment. In a third condition, participants began the trial with a moving start along the pointer in the intended direction; this condition yielded performance equivalent to that of the physical alignment condition. The results of this experiment imply that physical/mechanical factors are more effective in providing cues to initial orientation than are static explicit perceptual cues.

The totality of our findings point to nonperceptual factors, especially motor or biomechanical factors associated with movement, as the primary determinants of veering behavior. The four key findings contributing to this argument are: (a) history of visual experience does not influence veering behavior; (b) psychophysical thresholds for path curvature detection are not correlated with
veering behavior, although perceptual limitations of the motor system reveal a connection between curvature detection and veering behavior; (c) explicit perceptual cues for intended walking direction are less effective than physical alignment without an explicit perceptual cue; and (d) a simple stochastic model of walking behavior, assuming only stepping noise and no perceptual feedback, can account for the major features of our veering data.

Previous proposals for the origins of veering focused exclusively on causes that would create systematic deviations in walkers' trajectories away from a straight line. The most comprehensive review of these proposals is in Guth and LaDuke (1994), in which they describe a variety of physical asymmetries, biomechanical asymmetries and spiraling behavior and conclude that none of the historic proposals are plausible because the veering tendency is not always systematic. Our proposal is the first viable explanation of veering phenomena and the only one that can handle both systematic and unsystematic aspects of veering.

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## Appendix A

## A Random Walk Model for Walking Without Vision

The following Matlab code executes simulated walking trials. The parameters are fully adjustable, including the six mentioned walking parameters as well as the number of trials and number of steps per simulated trial.

```
% Matlab code simulation for walking without vision
nsims = 20; % n simulations
nsteps = 20; % n steps
in = [-1.152 2.182].*pi/180; % orientation (mu,sd) (degrees to radians)
ro = [ 2.024 0.328].*0.3048; % step length (mu,sd) (feet to meters)
th = [-0.122 1.332].*pi/180; % bias (mu,sd) (degrees to radians)
cumth = cumsum([normrnd(in(1),in(2),nsims,1)...
        normrnd(th(1), th(2),nsims,nsteps)],2);
steps = normrnd(ro(1),ro(2),nsims,nsteps);
cumx = cumsum([ones(nsims,1).*0 cos(cumth(:, 2:end)).*steps],2);
cumy = cumsum([ones(nsims,1).*0 sin(cumth(:, 2:end)).*steps],2);
plot(cumx',cumy','. :');
```


## Appendix B

## Relating Curvature Detection to Perceptual Error in Orientation Change Processing

We can estimate a perceptual orientation change threshold from the curvature detection data as follows: First, we convert the threshold curvature radius into a threshold orientation change per step. Then, we convert this threshold into an equivalent orientation change noise, a variance that characterizes the amount of error contained in the perceptual estimate of orientation change per step.

The orientation change $\gamma$ that occurs at each step of length $s$ when walking a circle of radius $R$ can be found, approximately, by division of the circumference of the circle into $N=2 \pi R / s$ steps of length $s$. Then, $\gamma=\frac{360}{N}=\frac{360 \cdot s}{2 \pi R}=57.3 \frac{s}{R}$ 。

This result holds when the step length is much smaller than the circumference, and this result is valid up to three decimal places for the radii and step lengths used in this article.

Assuming that the walker's perceived orientation change at a given step is represented by an internal variable $\alpha$ corrupted by Gaussian noise with zero mean and variance $\sigma_{\text {perceived }}^{2}$, then we can use a simple signal detection theory argument to relate the threshold $\gamma$ to the variance of this noise. The two-alternative forced choice (two-AFC) error rate for detection of an orientation change from one step on the
basis of $\alpha$ is given by (Green \& Swets, 1974): $\frac{\alpha^{2}}{\sigma_{\text {perceived }}^{2}}=Z\left(P_{\mathrm{c}}\right)^{2}$, where $Z\left(P_{\mathrm{c}}\right)$ is the $Z$ score of the probability correct $P_{\mathrm{c}}$, which is 0.9 for our curvature experiment. At threshold, $\alpha=\gamma$, so that $\sigma_{\text {perceived }}=\frac{\gamma}{Z\left(P_{\mathrm{c}}\right)}=\frac{57.3 \mathrm{~s}}{R Z(0.9)}=\frac{57.3 \cdot 2}{R \cdot 1.28}=\frac{89.5}{R}$.

For a threshold of $24.48 \mathrm{~m}, \sigma_{\text {perceived }}$ is equal to $1.12^{\circ}$. However, in our task, walkers make six to eight steps before they must make a decision. If the decision is made on the basis of measurements at more than one step, then the estimate above is a lower bound. For instance, if the decision is made on the basis of an average of $\alpha$ over $T$ steps and the corrupting noise is independent at each step, then $\frac{\sigma_{\text {perceived }}}{\sqrt{T}}=\frac{89.5}{R}$.

For example, if participants pool over the equivalent of two steps, then $\sigma_{\text {perceived }}=\sqrt{2} \cdot \frac{89.5}{R}$, which is $1.58^{\circ}$ for $R=24.48 \mathrm{~m}$.

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